Abstract:

We consider the problem of List Update one of the most fundamental problems in online algorithms and competitive analysis. Informally, we are given a list of elements and requests for these elements that arrive over time. Our goal is to serve these requests, at a cost equivalent to their position in the list, with the option of moving them towards the head of the list. Sleator and Tarjan introduced the famous "Move to Front" algorithm (wherein any requested element is immediately moved to the head of the list) and showed that it is 2-competitive. While this bound is excellent, the absolute cost of the algorithm's solution may be very large (e.g., requesting the last half elements of the list would result in a solution cost that is quadratic in the length of the list). Thus, we consider the more general problem wherein every request arrives with a deadline and must be served, not immediately, but rather before the deadline. We further allow the algorithm to serve multiple requests simultaneously - thereby allowing a large decrease in the solutions' total costs. We denote this problem as List Update with Time Windows. While this generalization benefits from lower solution costs, it requires new types of algorithms. In particular, for the simple example of requesting the last half elements of the list with overlapping time windows, Move-to-Front fails. In this work we show an $O(1)$ competitive algorithm. The algorithm is natural, but the analysis is a bit complicated, and a novel potential function is required.

Thereafter we consider the more general problem of List Update with Delays in which the deadlines are replaced with arbitrary delay functions. This problem includes as a special case the prize collecting version in which a request might not be served (up to some deadline) and instead suffers an arbitrary given penalty. Here we also establish an $O(1)$ competitive algorithm for general delays. The algorithm for the delay version is more complex and its analysis is significantly more involved.

Joint work with Shahar Lewkowicz and Danny Vainstein