Abstract:

Traced back to early 1960s, the research on finite-dimensional variational inequality (VI) problems began to evolve into a specific branch of mathematical programming and has since become a theoretical cornerstone in solving a diverse array of equilibrium-related problems, such as economic equilibrium, traffic equilibrium, and the general Nash equilibrium. The general mathematical formulation of VI problem encompasses various models such as optimization, saddle point problem, equation system, complementarity problem, and it is particularly suitable for solving the problems that necessitate the fusion of some of these models. For monotone VI problems, we introduce two classic methods: the (gradient) projection method and the extra-gradient method. In terms of iteration complexity, the extra-gradient method has proved to be an optimal method and emerged as the basis of a large variety of subsequent algorithms tailored for VI problems. This foundation establishes the connection to the contemporary challenges that VI researchers deal with. These challenges revolve around devising more efficient algorithms, both in practice and in theory, given specific problem context, including stochasticity, variance reduction, non-monotonicity, high-order derivative information, among others.